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Translated by L.K.

PMM U.S.S.R., Vol.47, No.4, pp.575-576, 1983
Printed in Great Britain
0021-8928/83 $10.00+00

UDC 539.3.534.1

ON THE REMAINDER TERMS IN THE FORMULAS FOR THE FREQUENCY DISTRIBUTION OF SHELL OSCILLATIONS*

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The frequency distribution of free oscillations of thin elastic shells in vacuo and in contact with a liquid, is studied. Estimates for the remainder terms in the asymptotic formulas for the oscillation frequency distribution are substantially improved. In the case of a shell in contact with a liquid, the second term of the asymptotics is separated.

Free oscillations of a thin elastic shell are described by a system of three differential equations in terms of the displacements /1/

$$(1/12h^2N + L)\,u = \lambda u,\quad \lambda = (1-\sigma^2)\,\rho E^{-1}\omega^2 \tag{1}$$

The vector function $u(\alpha,\beta)$, $(\alpha,\beta)\in G$ satisfies certain selfconjugate boundary conditions at the shell boundary, h is the shell thickness (small parameter), λ is the spectral parameter and ω is the natural shell oscillation frequency. The remaining notation is taken from /1/.

Let $n_h(\lambda)$ be the spectrum distribution function of problem (1) (equal to the number of eigenvalues less than the given λ). Using the variational method as $h\to+0$ we obtain /1/ the asymptotic formula

$$n_h(\lambda) = h^{-1}\,[c_0(\lambda) + O(h^{\varkappa})] \tag{2}$$

$$c_0(\lambda) = \frac{\sqrt{3}}{4\pi^2}\iint\limits_G\int\limits_0^{2\pi} \operatorname{Re}\,(\lambda - \Omega(\theta,\alpha,\beta))^{1/2}\,d\theta\,dS$$

$$\Omega(\theta,\alpha,\beta) = (1-\sigma^2)\,[R_1^{-1}(\alpha,\beta)\sin^2\theta + R_2^{-1}(\alpha,\beta)\cos^2\theta]^2$$

where $\varkappa$ is a positive number. A rough lower estimate was given for it in /1,2/.

By improving the variational technique, we succeeded in showing that when

$$\lambda > \sup\Omega(\theta,\alpha,\beta),\quad \theta\in[0,2\pi],\ (\alpha,\beta)\in G \tag{3}$$

formula (2) holds with $\varkappa = 1/_2 - \varepsilon$, for arbitrarily small positive ε.

If condition (3) does not hold, then the value of $\varkappa$ decreases and depends on the amount of "degeneration" of the function $q = \lambda - \Omega(\theta,\alpha,\beta)$. For example, if $q^{-1/2}$ is integrable, $\theta\in[0, 2\pi]$, $(\alpha,\beta)\in G$ (simple degeneration), formula (2) holds for $\varkappa = 5/22$.

In the same manner we can improve the estimate of the remainder in the problem of free oscillations of a shell in contact with an ideal compressible liquid /3/. In this problem we add to the right-hand side of the third equation of (1) the term $-h^{-1}\rho_f\rho_0^{-1}\lambda\varphi\,|_S$ (the inertia of the liquid). The potential $\varphi(x,y,z)$ of the displacement of the liquid occupying a finite volume V, satisfies the Helmholtz equation

$$\Delta\varphi + k_0\lambda\varphi = 0,\quad (x,y,z)\in V,\quad k_0 = \frac{E}{(1-\sigma^2)\,\rho_0 c_f^2}$$

*Prikl. Matem. Mekhan., Vol.47, No.4, pp.703-704, 1983

(c_f is the speed of sound in the liquid). The volume V is bounded by the middle surface of the shell S and two flat bases S_1 and S_2 perpendicular to the z-axis. We assume that

$$\frac{\partial\varphi}{\partial z}\Big|_{S_1}=0,\ \varphi\Big|_{S_2}=0,\ \frac{\partial\varphi}{\partial n}\Big|_{S}=-u_3$$

The following asymptotic formula holds for the shell-liquid system spectrum distribution function $n_h(\lambda)$ (the complete formulation of the problem is given in /3,4/):

$$n_h(\lambda)=h^{-3/2}[c_1(\lambda)+c_2(\lambda)h^{1/4}+o(h^{1/4})] \tag{4}$$

$$c_1(\lambda)=\frac{\Lambda^{3/2}}{4\pi}S_0,\ \Lambda=12\lambda\frac{\rho_f}{\rho_0}$$

$$c_2(\lambda)=-\frac{3\pi^2}{5}\Lambda^{-1/4}\iint\limits_G\int\limits_0^{2\pi}(\Omega-\lambda)\,d\theta dS$$

(S_0 is the shell surface area) with o-term uniform in λ for $0<a\leqslant\lambda\leqslant b$.

Note that the problem of strengthening the remainders in the formulas given /1/ and /3/ is the subject of the paper by S.Z. Lebendorskii*.

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